Casson-Freedman 理論研究会　の資料

Freedman 原論文 (J.D.G 17 (1982))
１０章 の解説

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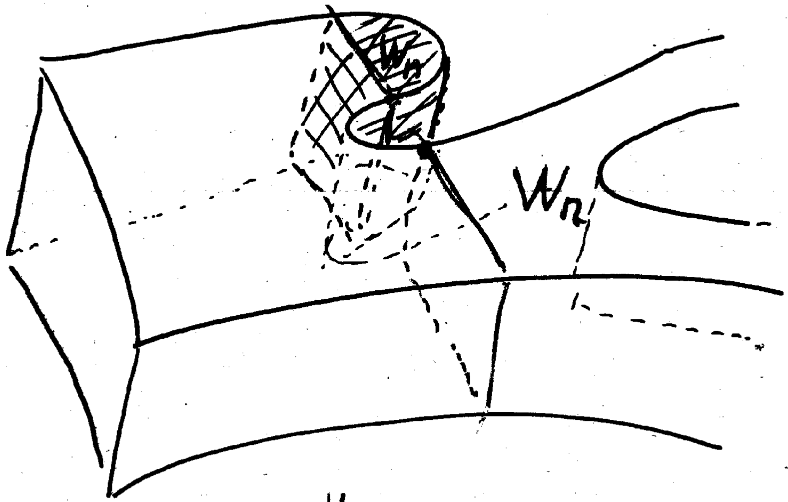
２００９年　１０月１７日～２０日
Casson-Freedman 理論研究会　の資料として
講演で用いられた ＯＨＰシート を複写し、電子化させていただきました.

２００９年１０月.
電子化　山田裕一（集会世話人の１人）

① remove the compact components from W_n



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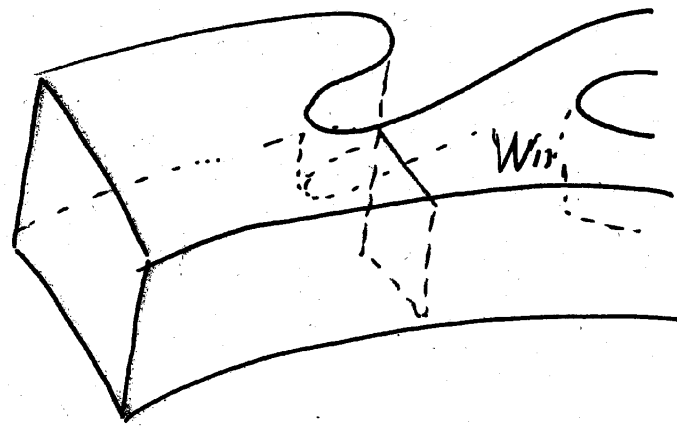
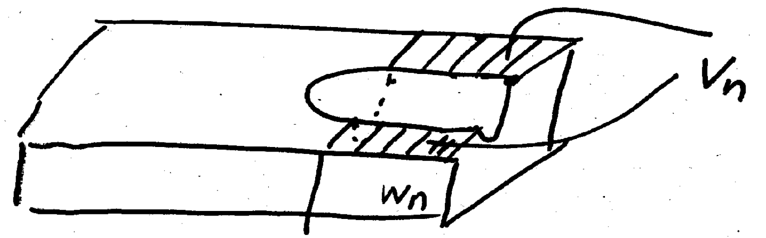


② If $\mathrm{Ker}\ \pi_0 V_n^{(1)} \to \pi_0 W_n \neq 0$

$\exists\, \gamma \subset V_{n-1}$ arc connecting different components of V_n

(suppose $\pi_0 V_{n-1}^{(1)} \xrightarrow{\cong} \pi_0 W_{n-1}$)



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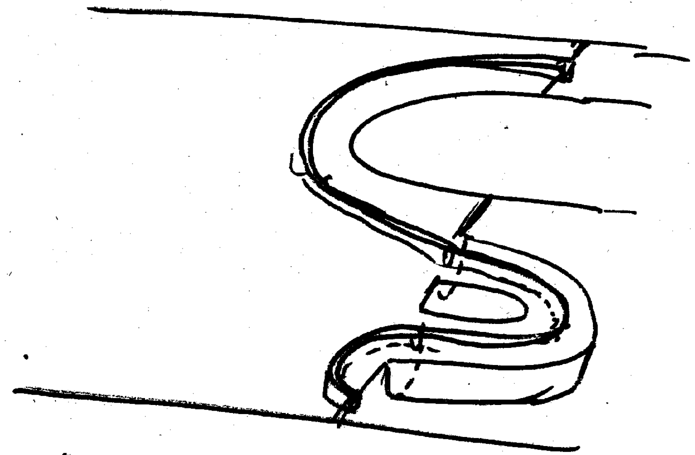


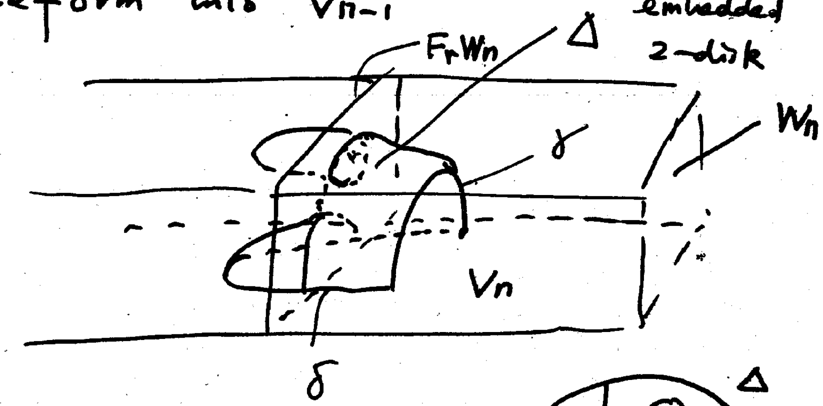
↑ handle exchange

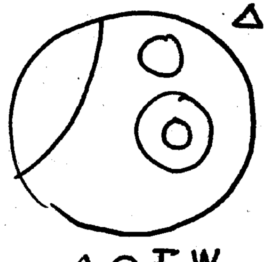
$\Rightarrow \quad \pi_0 V_n^{(1)} \xrightarrow{\sim} \pi_0 W_n$

3°

$\gamma \neq 0 \in \pi_1(W_n, V_n^{(1)}) \Rightarrow$

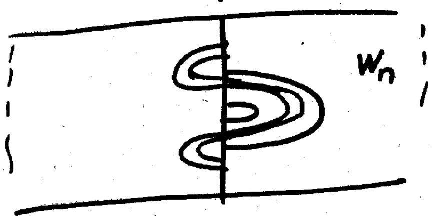
deform into V'_{n-1}

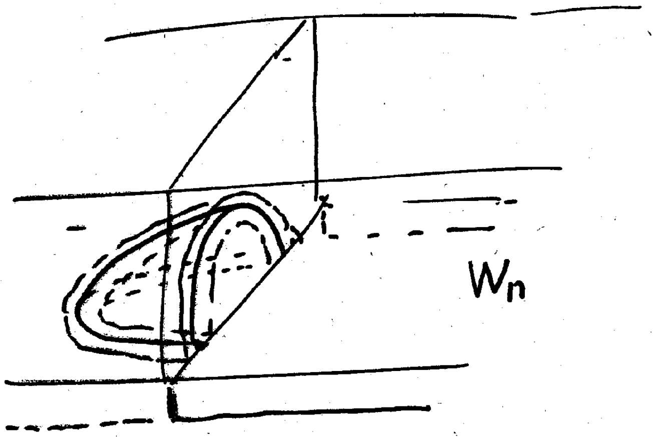




$\Delta \cap Fr\,W_n$

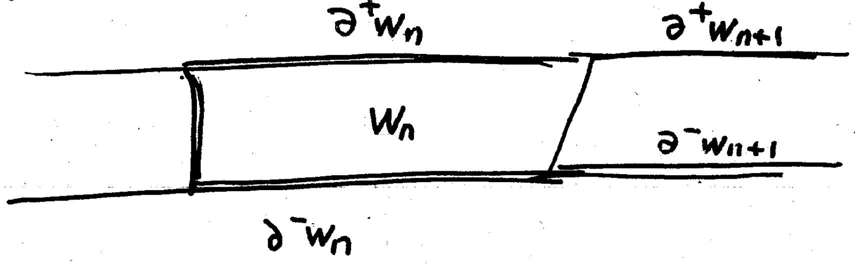
2-handle exchange





$\Rightarrow \quad \pi_1(W_n \; V_n^{(1)}) = 0$

4°

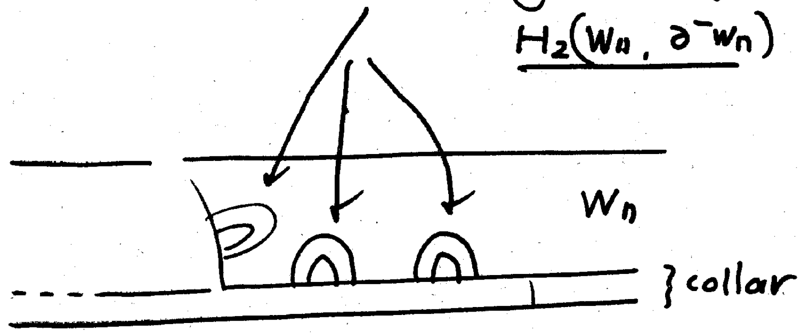


$\pi_1(W_n, \partial^- W_n) = 0 \Rightarrow H_1(W_n, \partial^- W_n) = 0$

$\pi_2(W_n, \partial^- W_n) \twoheadrightarrow H_2(W_n \; \partial^- W_n)$

finitely generated

subtract 2-handles generating $H_2(W_n, \partial^- W_n)$

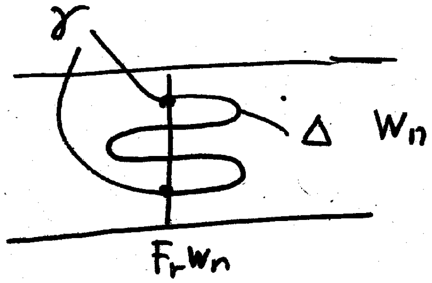


$\Rightarrow \quad H_2(W_n, \partial^- W_n) = 0$

5° $\gamma \neq 0 \in \pi_1 F_r W_n$

$\Rightarrow \gamma = \partial\Delta \subset \Delta \subset W_{n-1}$ embedded 2-disk

2-handle exchange along Δ



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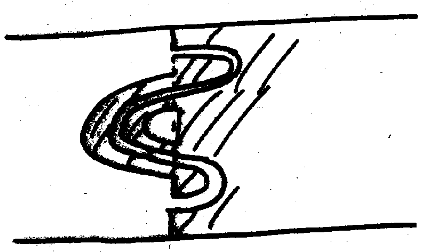

$\Rightarrow \pi_1 F_r W_n = 0$

$(\Rightarrow \pi_1 W_n = 0)$

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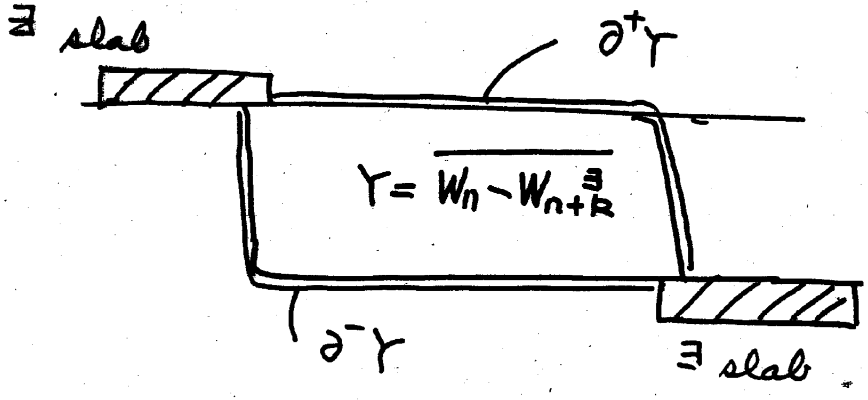


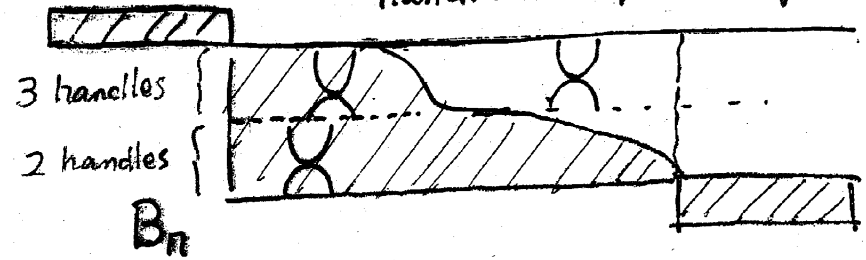
$Y \supset$ support of $H_3(W_3, \partial^- W_3)$

$Y_+ = Y \cup \exists$ slabs

slab $=$ $V^{(1)}$ の collar の一部

$\approx$ (compact 4 manifold with ∂) $\times I$

handle decomposition of Y



$B_n :=$ all 2-handles $\cup$ 3 handles generating $H_3(W_3, \partial^- W_3)$

$\cap$

Y

$\pi_1 \partial^{\pm} B_n \xrightarrow{\text{iso}} \pi_1 B_n$

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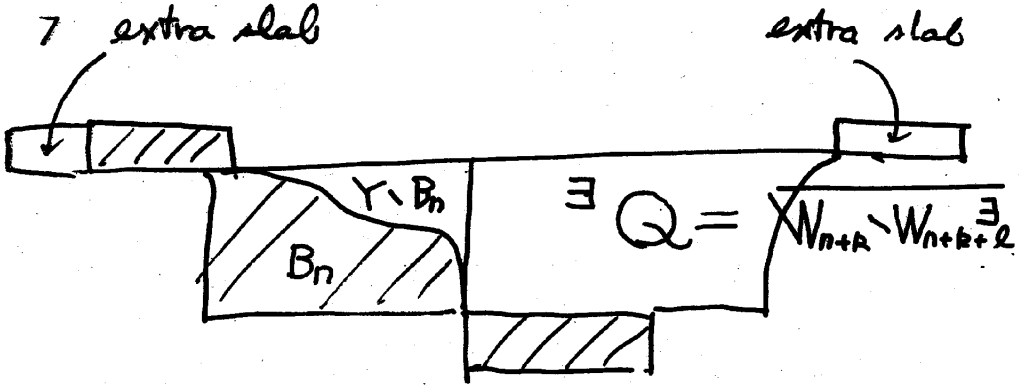
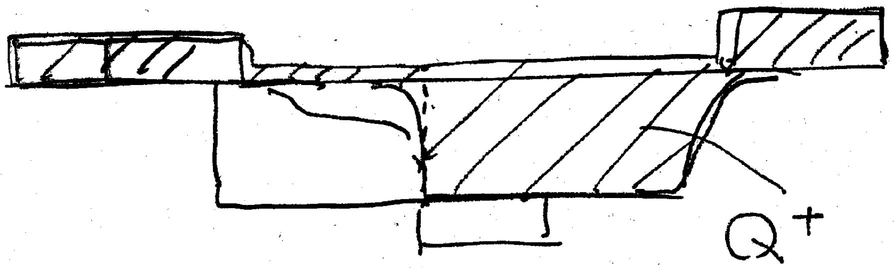




$$Q^+ = Q \cup {}^{\exists}\text{extra slabs}$$

s.t.

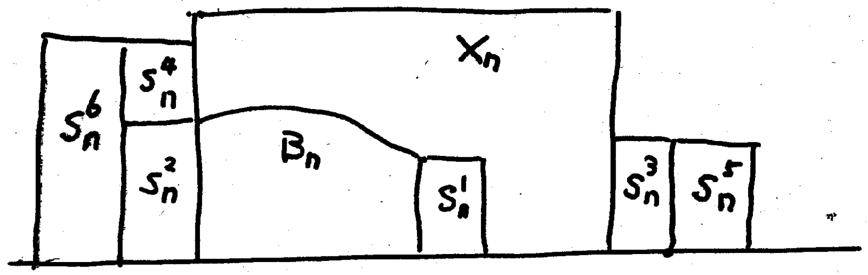
$$\pi_1 \partial^{\pm} Q^+ \xrightarrow{\cong} \pi_1 Q^+$$

$$X_n := (Y \setminus B_n) \cup Q^+$$

$$\Rightarrow \pi_1 \partial^{\pm} X_n \xrightarrow{\cong} \pi_1 X_n$$

8 add extra slabs

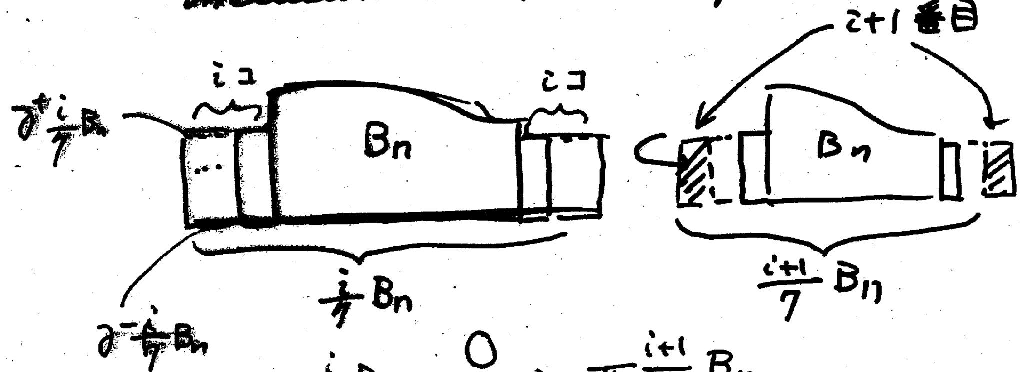
$$Z_n = B_n \cup X_n$$



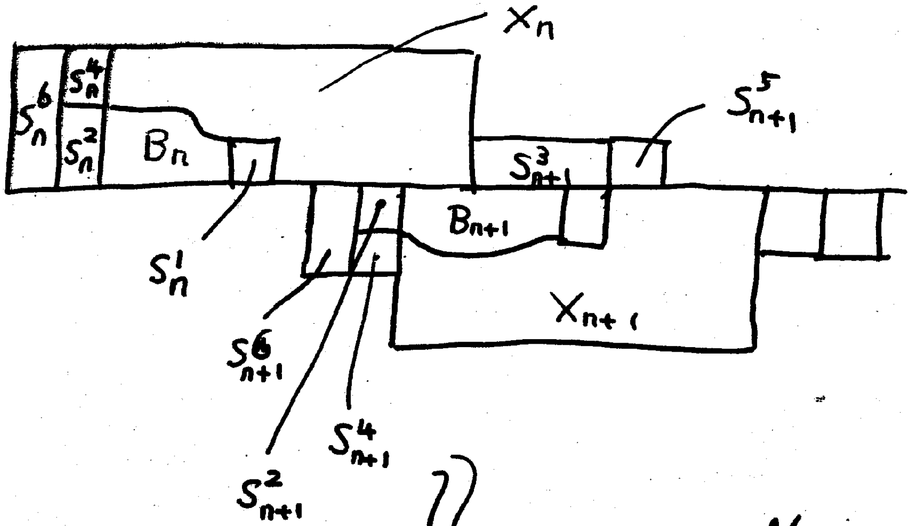
各 ${}^{\exists}S_n^i = {}^{\exists}\tfrac{1}{7}S_n^i \cup {}^{\exists}\tfrac{2}{7}S_n^i \cup \cdots \cup {}^{\exists}\tfrac{7}{7}S_n^i$

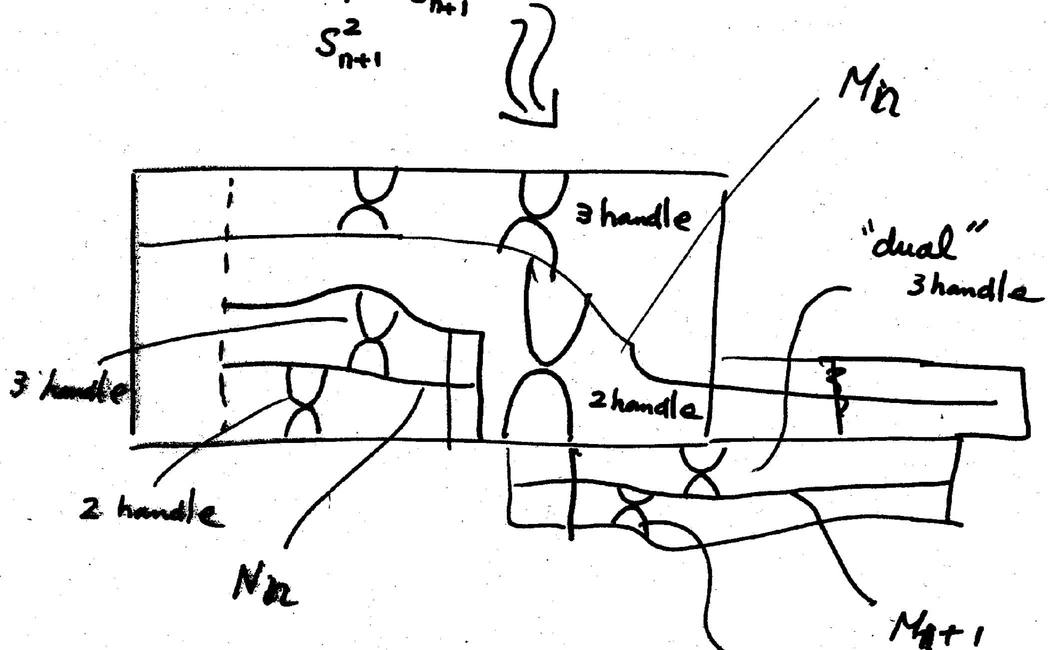
7個の slab の集合体

s.t. 左右に1個ずつ slab を加える度に inclusion は π_1 の 0 map を induce



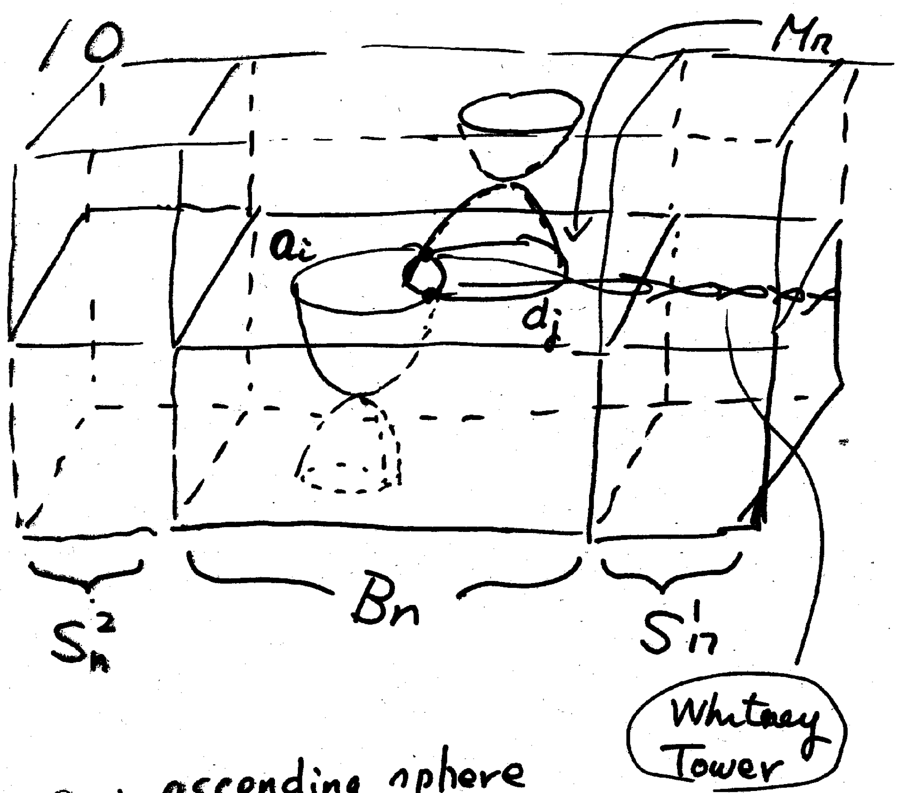
$$\begin{tikzcd}
\pi_1 \frac{i}{7}B_n \arrow[r, "0"] & \pi_1 \frac{i+1}{7}B_n \\
\pi_1 \partial^{\pm}\frac{i}{7}B_n) \arrow[u, "\cong"] \arrow[r, "0"] \arrow[ur, phantom, "\circlearrowright"] & \pi_1 \partial^{\pm}(\frac{i+1}{7}B_n) \arrow[u, "\cong"']
\end{tikzcd}$$





N_n mid level for B_n

M_n mid level for X_n



a_i : ascending sphere for the 2-handle

d_j descending sphere for the 3-handle

$a_i \cdot d_j = 0 \in \mathbb{Z}$ in M_n

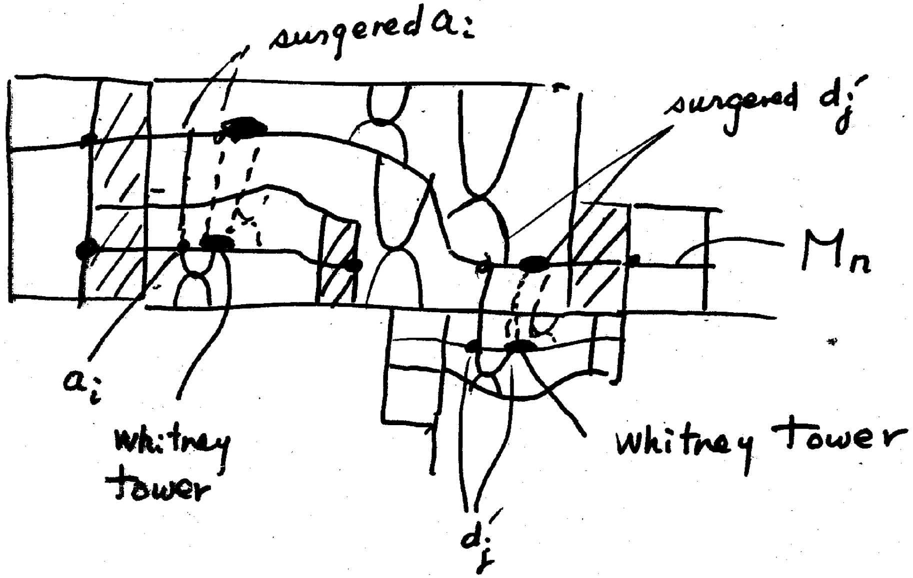
$\Downarrow$

$a_i \cdot d_j = 0 \in \mathbb{Z}[\pi_1(M_n^{\frac{1}{2}})]$ "with geometric dual spheres"

$\Downarrow$

$a_i \cap d_j = \{ p^+ \; p^- \; \cdots\cdots \}$

Cancelled by $\exists$ Whitney tower in M_n^1 (π_1-negligible)



A_i's : $\left\{\begin{array}{l}\text{ascending spheres for}\\ \text{2-handles in } X_n\end{array}\right\} \cup \left\{\begin{array}{c}\text{surgered}\\ a_i\end{array}\right\}$

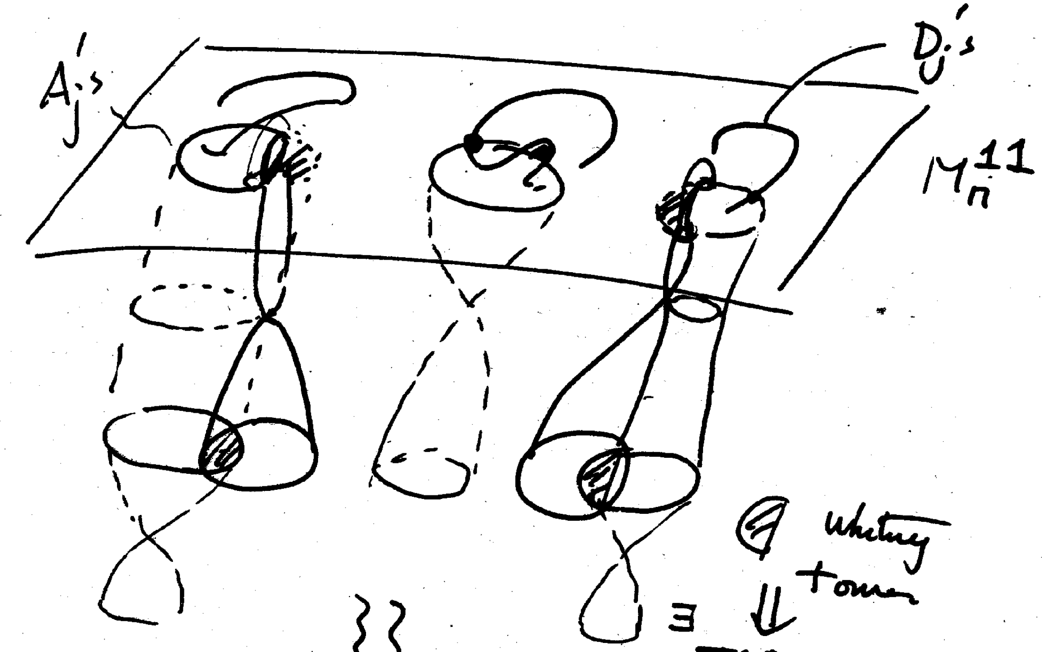
D_j's : $\left\{\begin{array}{l}\text{descending spheres for}\\ \text{3-handles in } X_n\end{array}\right\} \cup \left\{\begin{array}{c}\text{surgered}\\ d_j'\end{array}\right\}$

in $M_n^1 \doteq$ mid level of $X_n \cup S_n^4 \cup S_{n+1}^3$

$\Rightarrow$ $A_i \cdot D_j' = \delta_{ij}$ (after handle slides)

$\Rightarrow$ $\exists$ Whitney towers for "cancelling" intersections in the mid level of

$X_n \cup S_n^4 \cup S_{n+1}^3 \cup S_n^6 \cup S_{n+1}^5 = M_n^{11}$

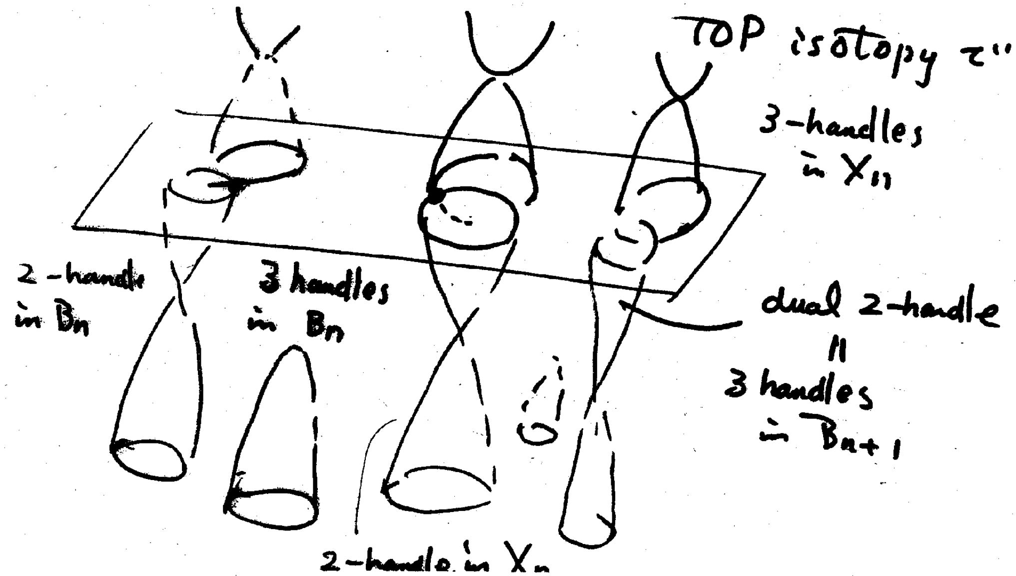


$\exists$ $\Downarrow$

TOP embedded Whitney disk

$\Downarrow$

TOP isotopy τ''



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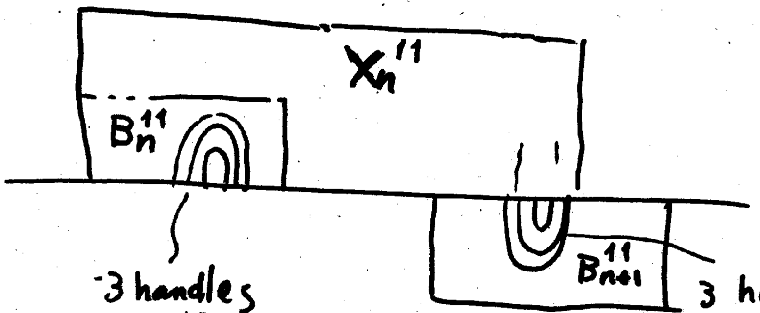
$Z_n'' = X_n'' \cup B_n''$



3 handles in B_n'' generating $H_3(W_n, \partial^- W_n)$

3 handles in B_{n+1}'' generating $H_3(W_{n+1}, \partial^+ W_{n+1})$

$\tilde{Z}_n = Z_n'' \setminus \{\cup \text{ 3 handles in } B_n''\}$
$\cup (\cup \text{ 3 handles in } B_{n+1}'')$

$\wr\wr$

$(\tilde{Z}_n \quad \partial^- \tilde{Z}_n \quad \partial^+ \tilde{Z}_n)$

2 & 3 handles ···· cancell

$\Rightarrow$ TOP product