２００７年９月１１日
低次元幾何学 と 無限次元幾何学

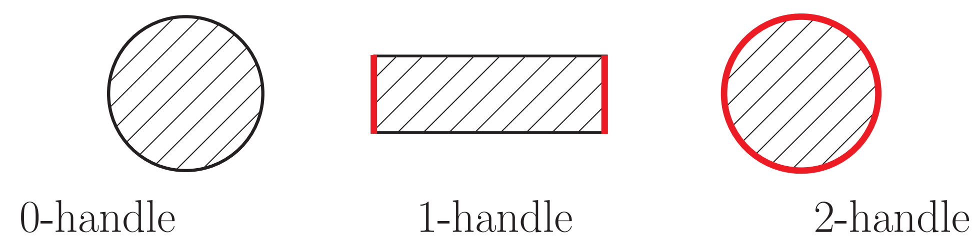
Kirby Calc. 入門

山田 裕一　　（電気通信大学）

1. [K] R. Kirby, “The topology of 4-manifolds”, Springer Lecture Notes **1374** (1989).
2. [GS] R. Gompf & A. Stipsicz, “4-Manifolds and Kirby Calculus”, (1999).

§1. Handle Decomposition

• 2-dim.



• 3-dim.

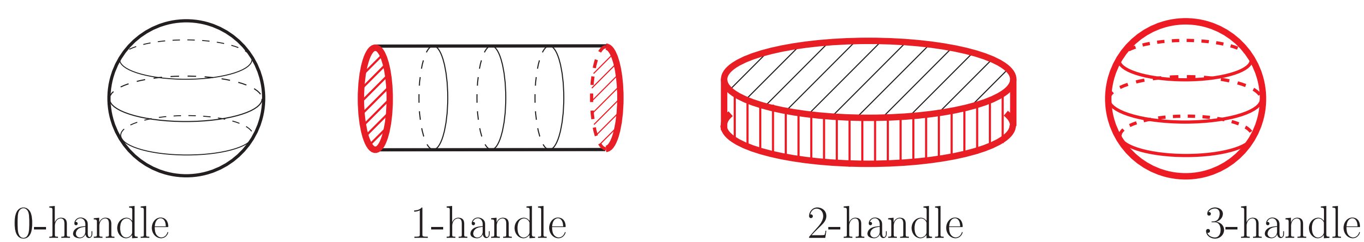


Figure 1-1 : 2-, 3-dim. handles

Why (and When) handles are useful?

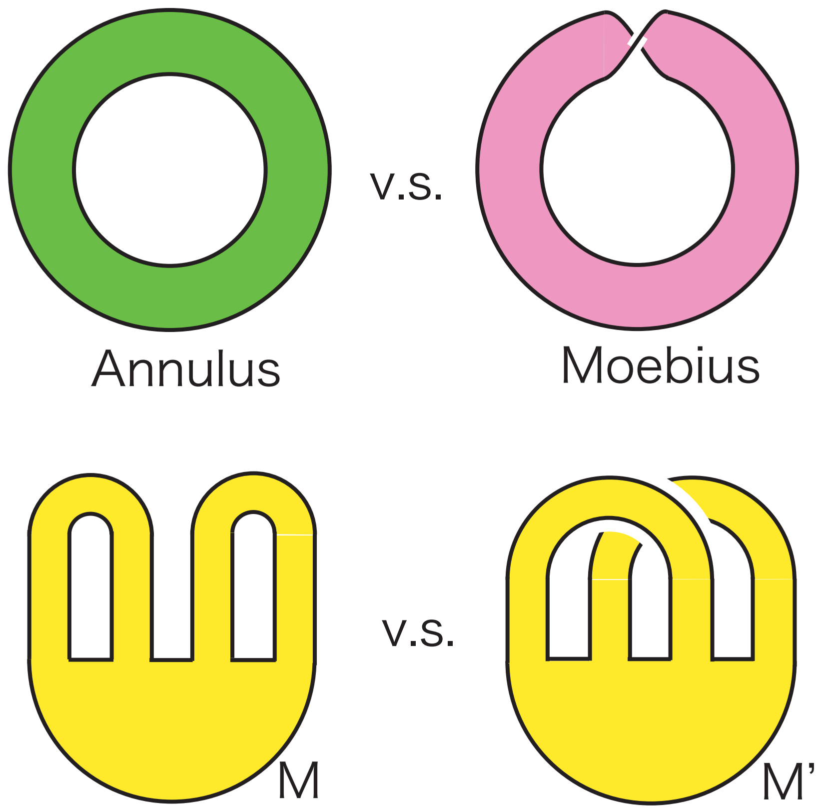
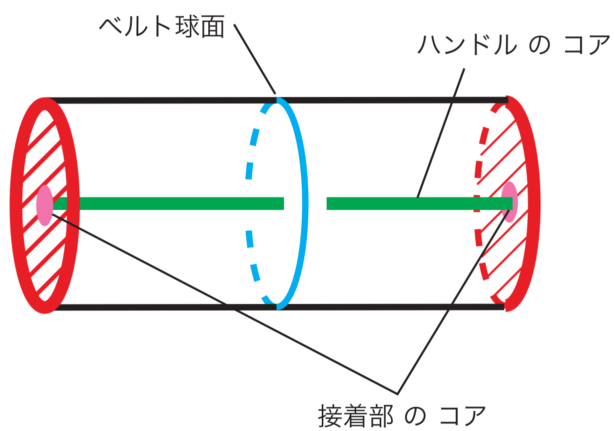


Figure 1-2 : homotopy eq. but not-homeo.

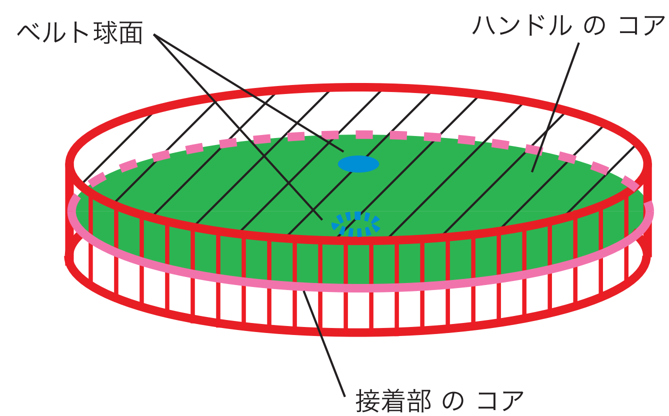
($M \simeq M'$ but $M \not\cong M'$, $\quad M \times [0,1] \cong M' \times [0,1]$)

Terminology of handle

n-dim. i-handle	$D^i \times D^{n-i}$		handle Core	$D^i \times \{o\}$
Attaching part	$\partial D^i \times D^{n-i}$			
Boundary part		$D^i \times \partial D^{n-i}$		
Attach Core	$\partial D^i \times \{o\}$		its *parallel*	$\partial D^i \times \{p\}$
Belt sphere		$\{o\} \times \partial D^{n-i}$		



1-handle



2-handle

Figure 1-3 : 3-dim. handles

1-handle is identifying the two balls

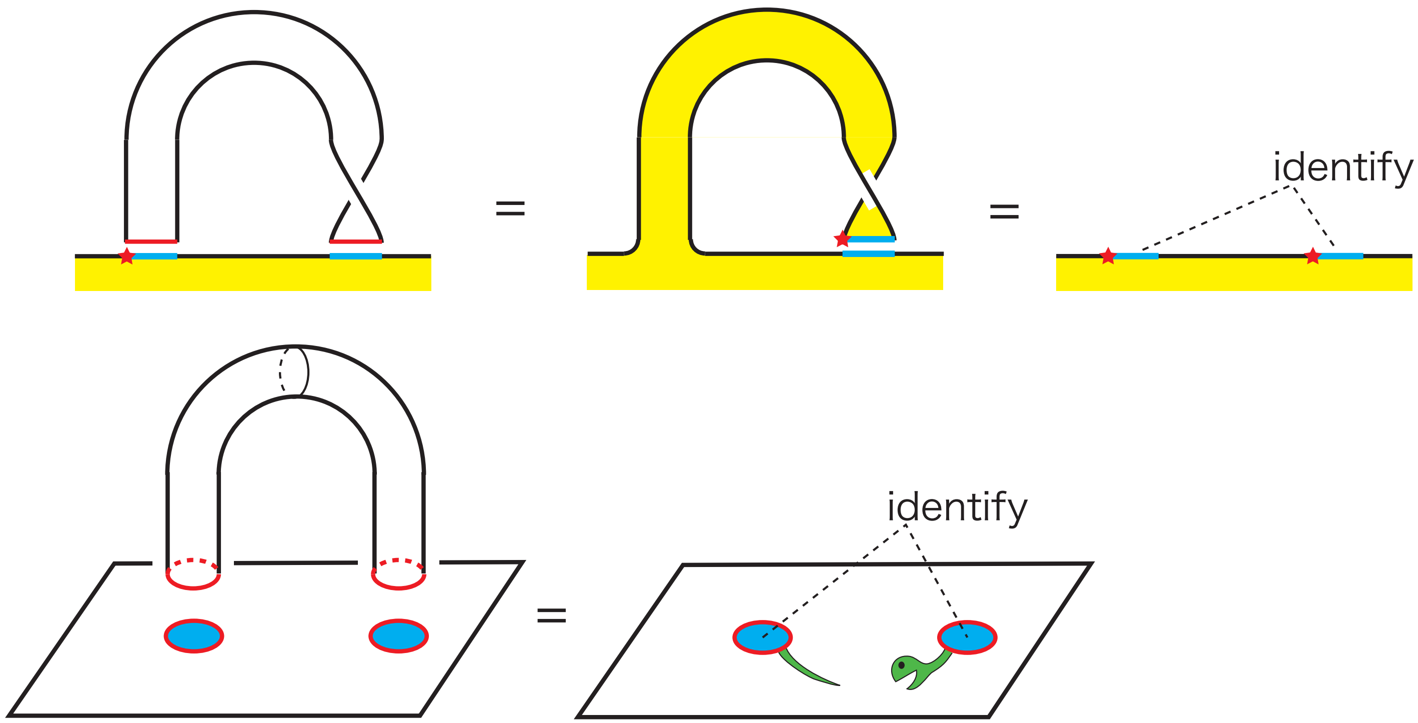


Figure 1-4 : 1-Handle

Only 1-handle can connect disjoint parts ...

Handle Slide (h_i^i slides over h_j^i)

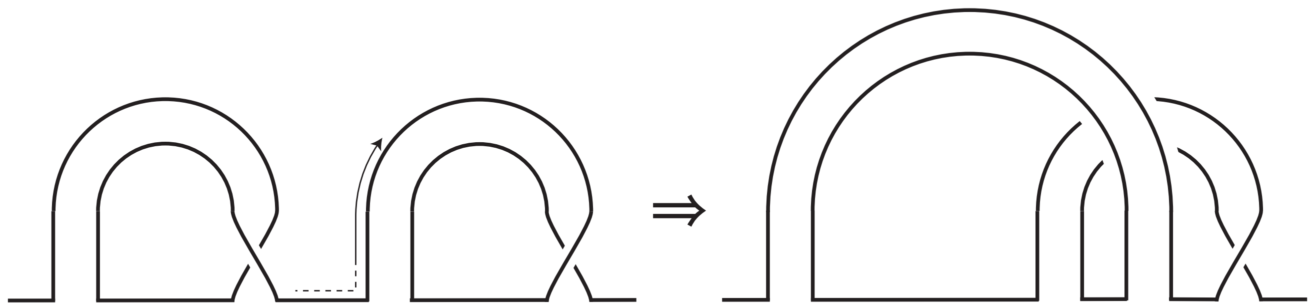


Figure 1-5 : Handle Slides

$$Mob \natural Mob \cong \text{punc}Kb, \qquad \mathbf{R}P^2 \sharp \mathbf{R}P^2 \cong Kb \text{ (i.e. } S^1 \tilde{\times} S^1)$$

Handle Cancel (h^i and h^{i+1} cancel, if "Belt$(h^i) \cap$ At.core$(h^{i+1}) = \{1\text{pt}\}$")

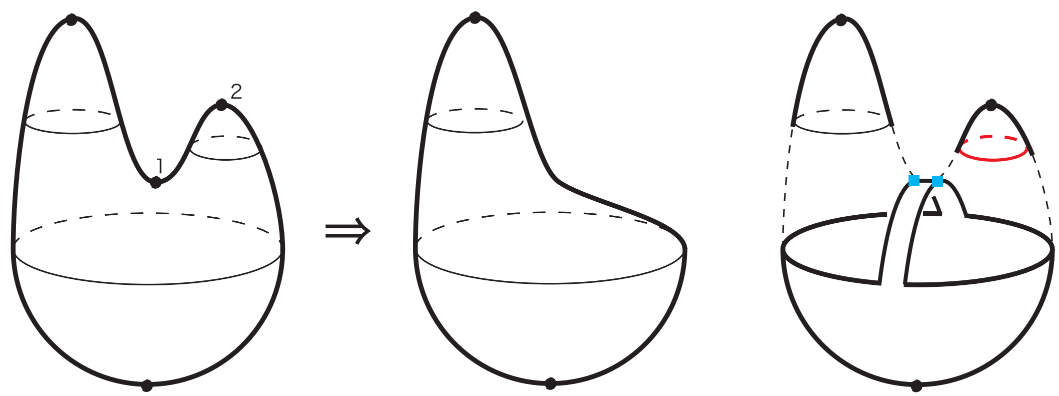


Figure 1-6 : Handle Cancel

i-handle presents i-chain, i-cycle, or i-homology

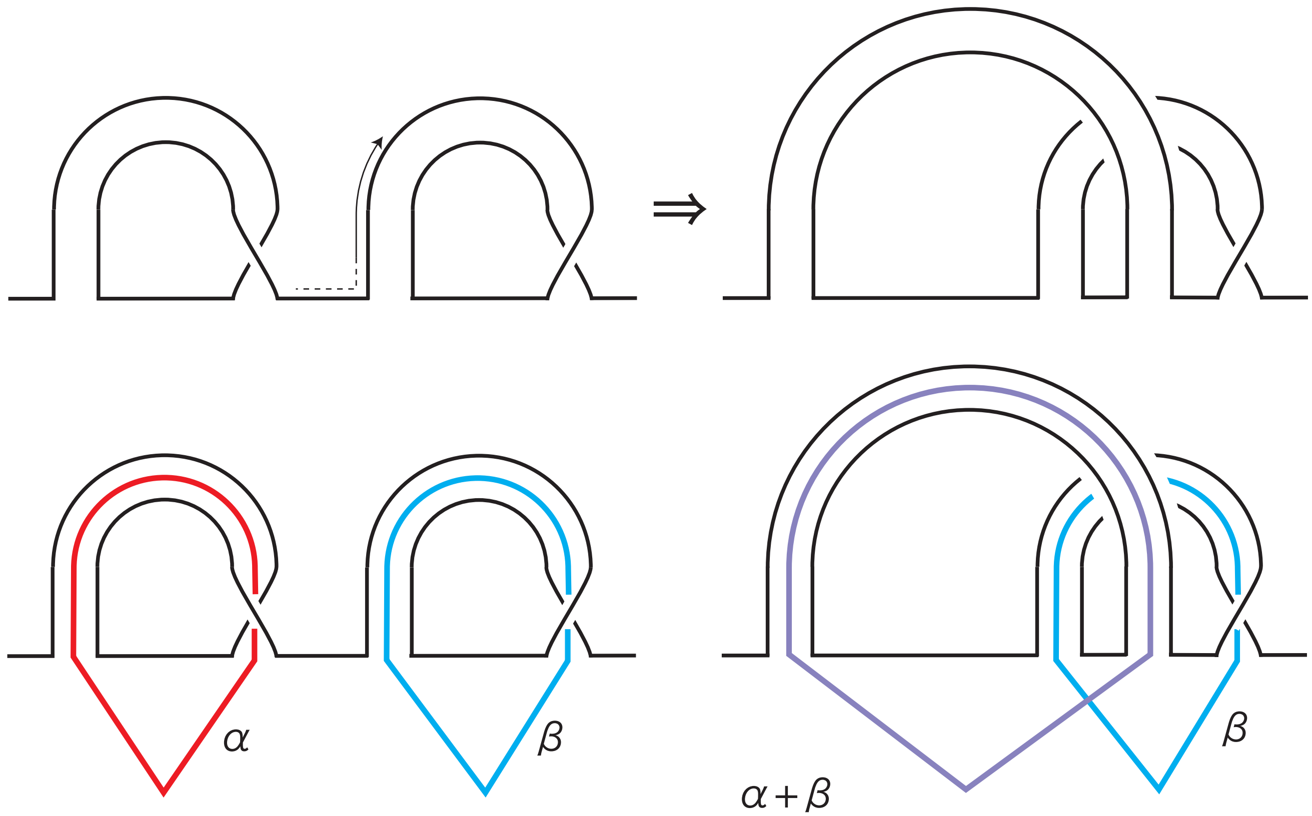


Figure 1-7 : Handle presents Homology

Handle slide is "+/− in homology"

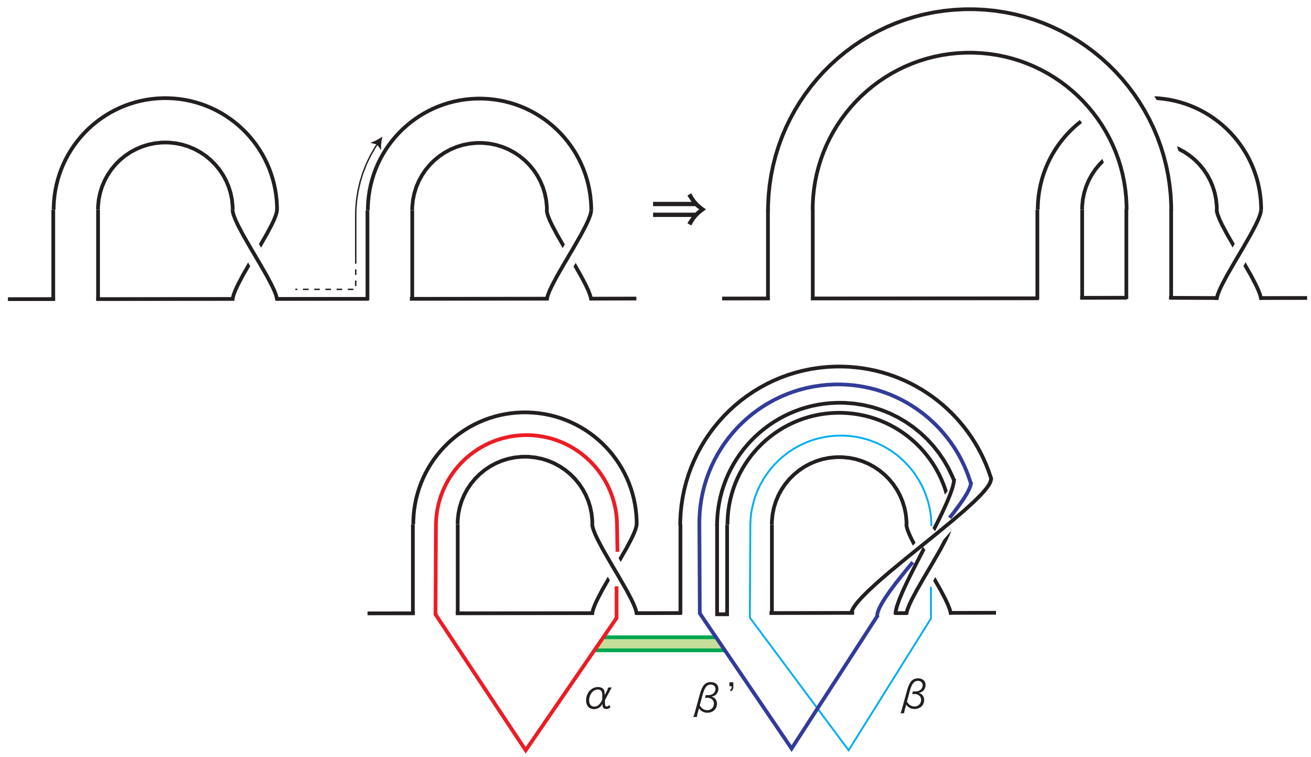
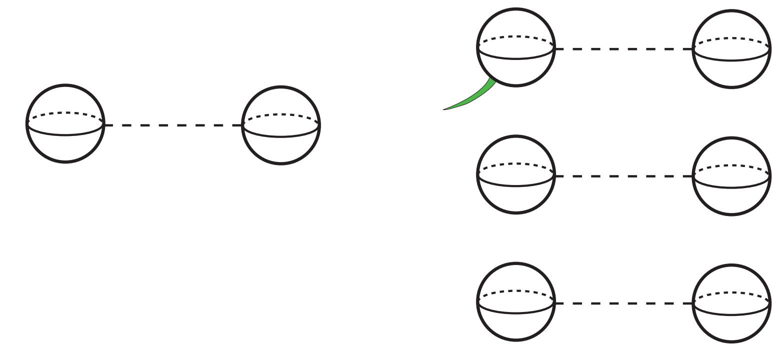


Figure 1-7'

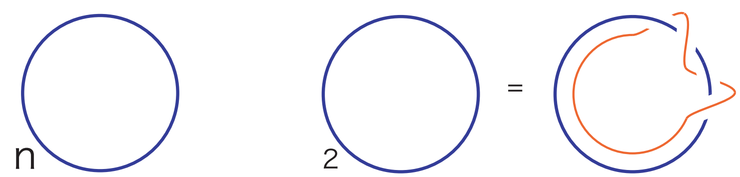
§2. 4-dim. case

ex1.



$S^1 \times D^3$ $\quad$ $\natural_3(S^1 \times D^3)$ (or $\sharp_3(S^1 \times S^3)$)

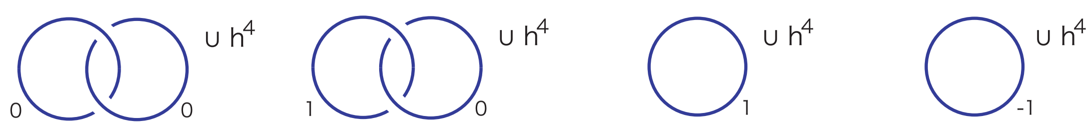
ex2.



$S^2 \times_n D^2 :=$「S^2 上の D^2 束 with $Z \cdot Z = n$」 $Z =$ zero section

$$\partial(S^2 \times_n D^2) \cong -L(n, 1).$$

ex3.



$S^2 \times S^2$ $\quad$ $S^2 \tilde{\times} S^2$ $\quad$ $\mathbf{C}P^2$ $\quad$ $\overline{\mathbf{C}P}^2$ $(-\mathbf{C}P^2)$

Figure 2-1 :

Blow-Up

$$\begin{array}{ccc} \mathbf{C}^2 \times \mathbf{C}P^1 & \supset & U := \{((z,w),[s:t]) \,|\, zt = ws\} \\ \downarrow_{pr.1} & & \downarrow \pi := pr.1|_U \\ \mathbf{C}^2 & & (z,w) \end{array}$$

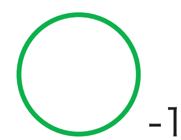
Note

(1) $\pi^{-1}(z,w) = ((z,w),[z:w])$ for $(z,w) \neq (0,0)$

(2) $\pi^{-1}(0,0) \cong \mathbf{C}P^1 = S^2$ $\quad (U \cong \text{punc}\overline{\mathbf{C}P^2})$

local coordinates of U

$$\begin{array}{ccccc} \mathbf{C}\times\mathbf{C} & \to & U & \leftarrow & \mathbf{C}\times\mathbf{C} \\ (z,t) & \to & ((z,zt),[1:t]) & & \\ & & ((ws,w),[s:1]) & \leftarrow & (w,s) \\ (z,t) & & \to & & (zt,\frac{1}{t}) \end{array}$$



In $\mathbf{C}^2$, $(z,w) = (z,zt) = (ws,w)$.

2-handle slides Slide h_2 over h_1

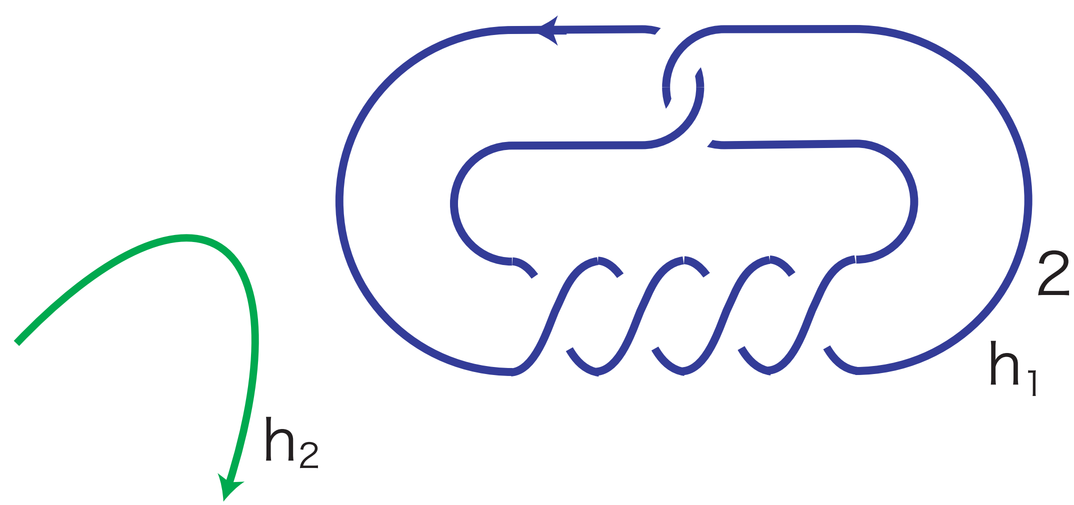


Figure 2-2 :

2-handle slides

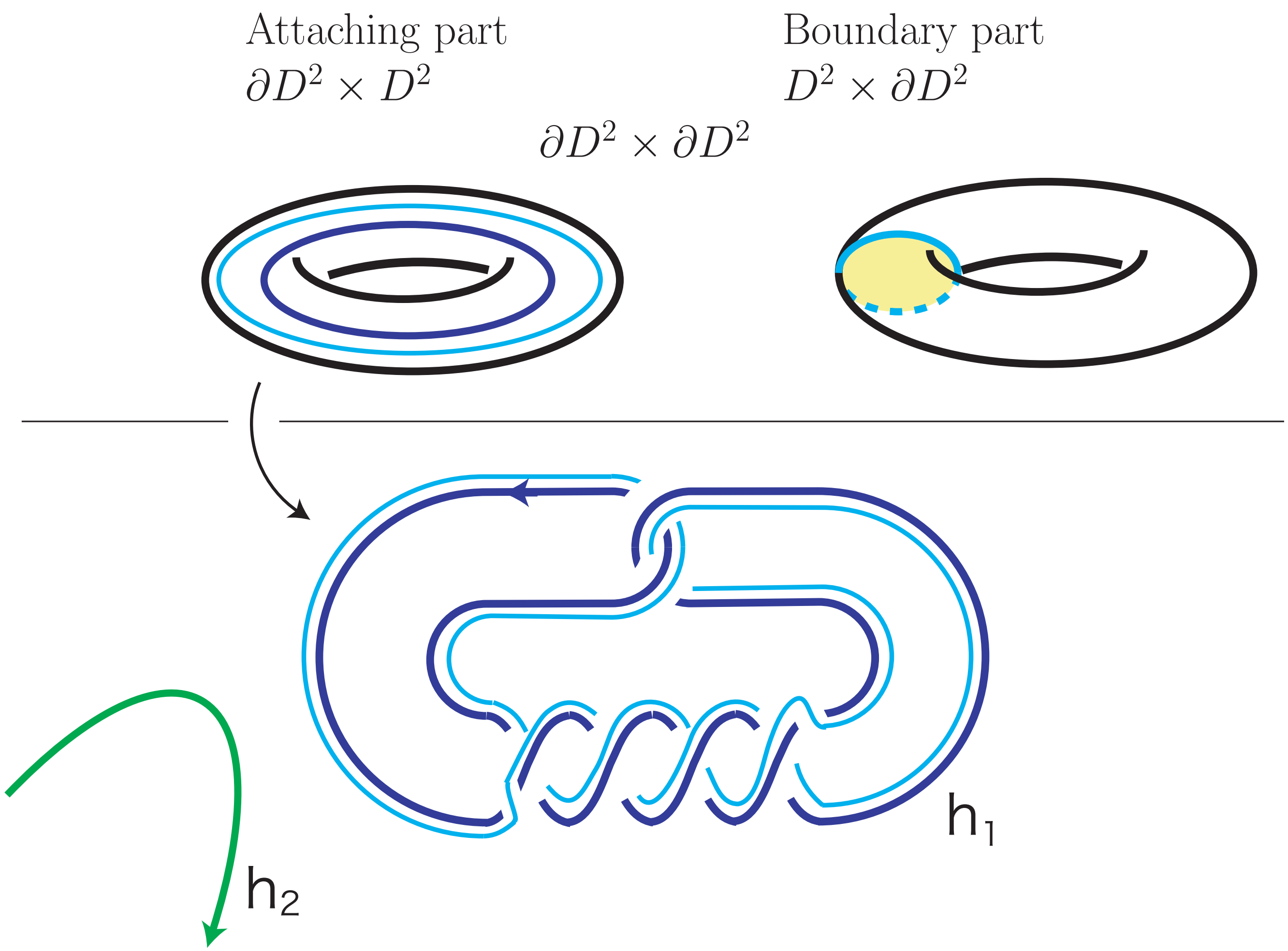


Figure 2-2(2) :

2-handle slides

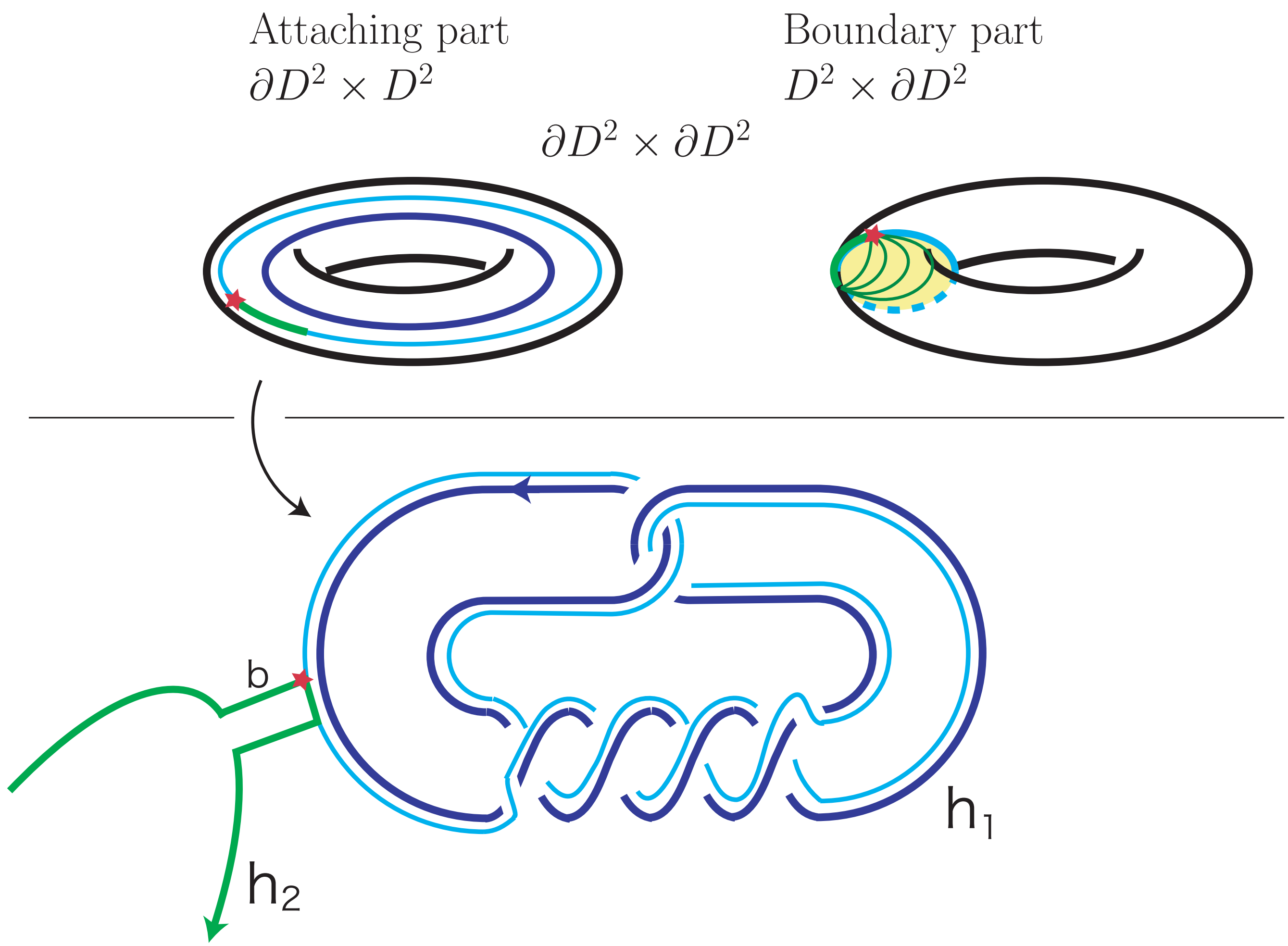


Figure 2-2(3) :

2-handle slides

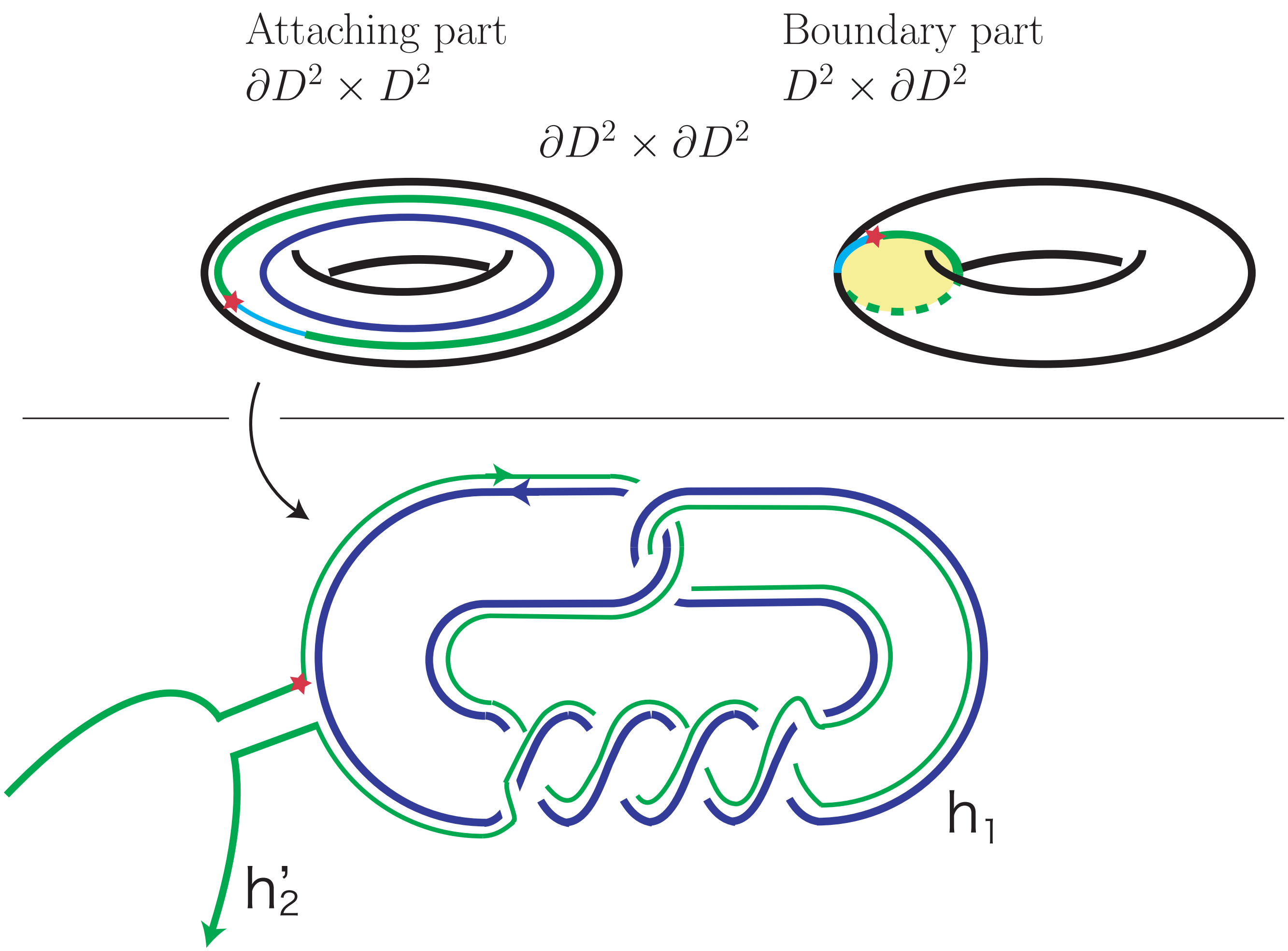


Figure 2-2(results) :

Framing of h'_2 is ....

Useful Formula F0: "A component with 0-framed meridian" can be taken out.

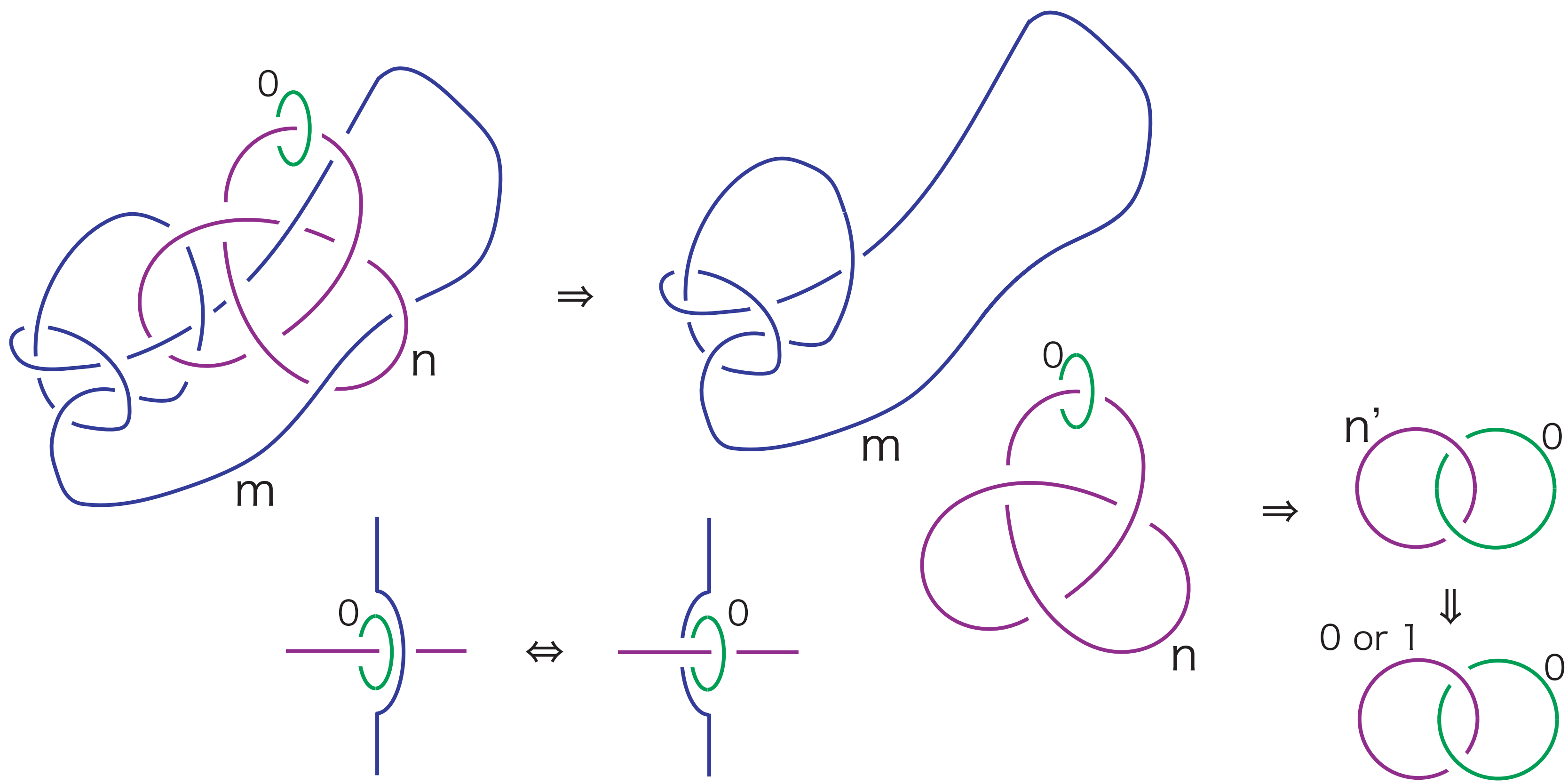
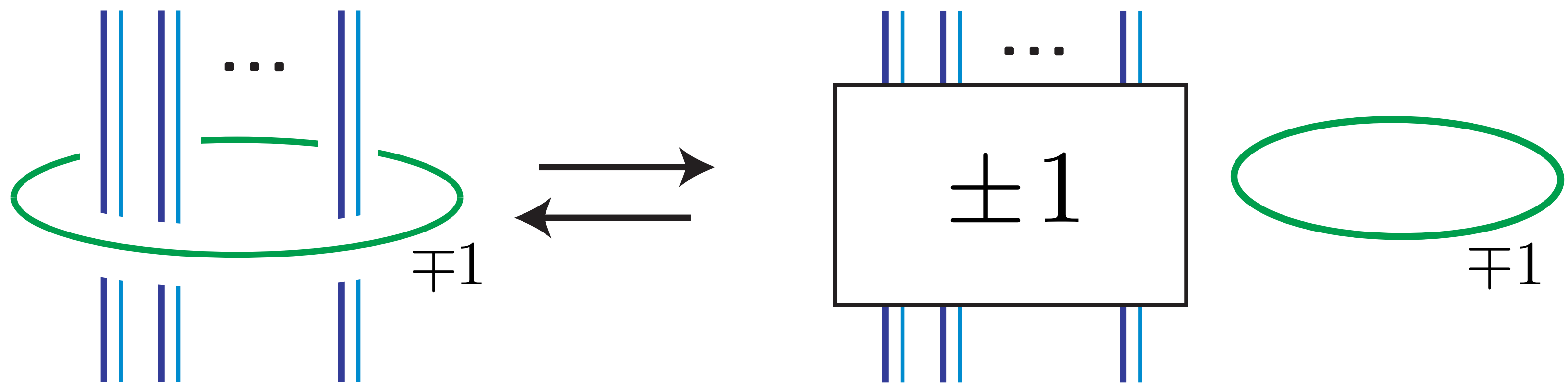
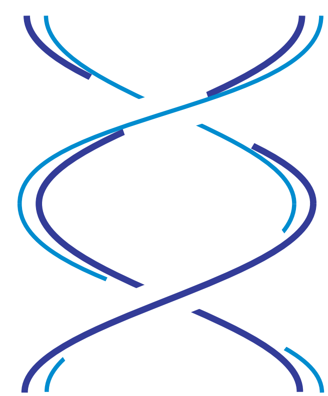


Figure 2-3 :

Useful Formula F1: related to Blow-up





Exercise

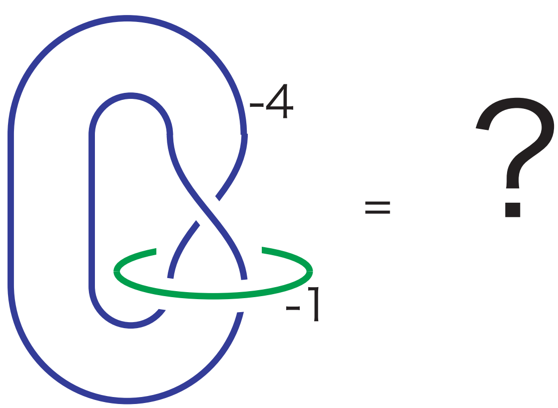
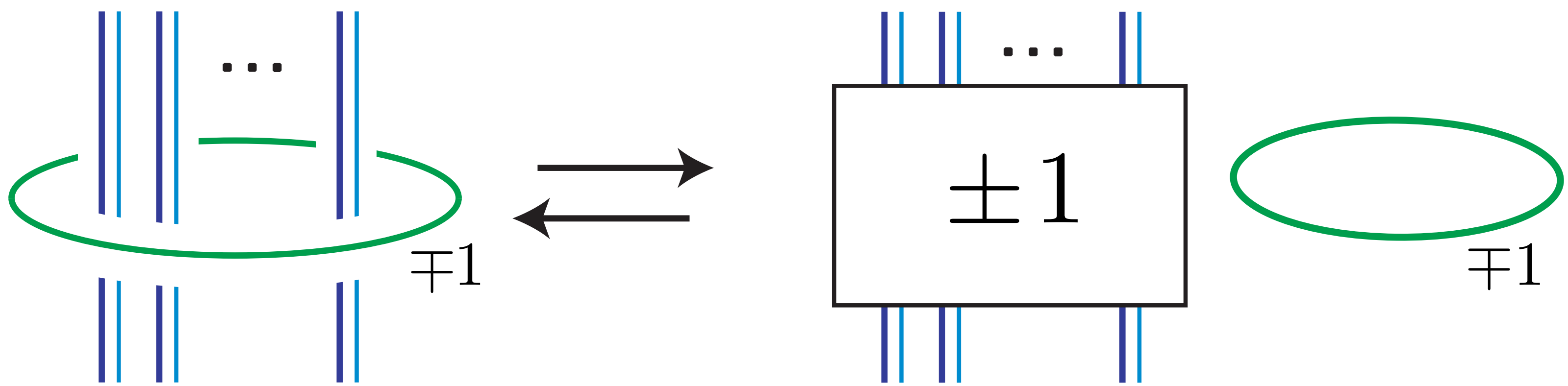
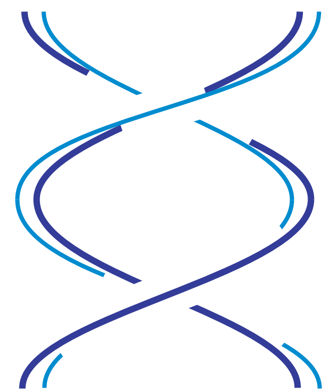


Figure 2-4 :

Useful Formula F1: related to Blow-up





Answer

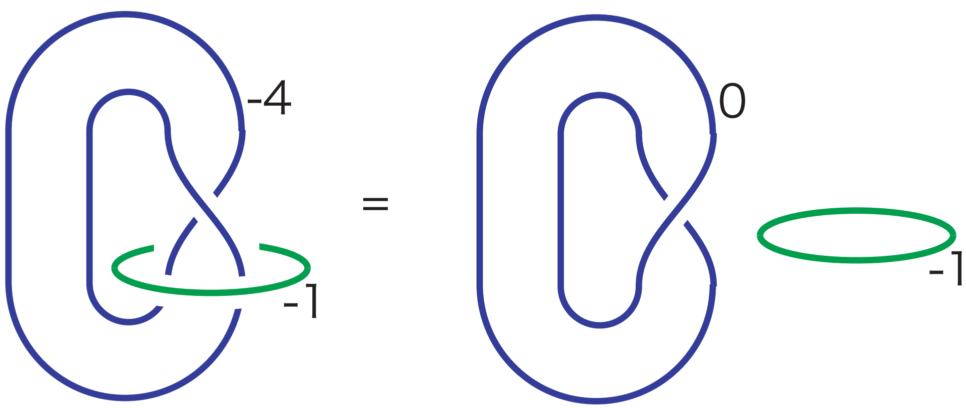


Figure 2-4a :

Intersection form = linking matrix.

$$E_8 = \begin{bmatrix} -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\mathbf{2} & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -2 \end{bmatrix}$$

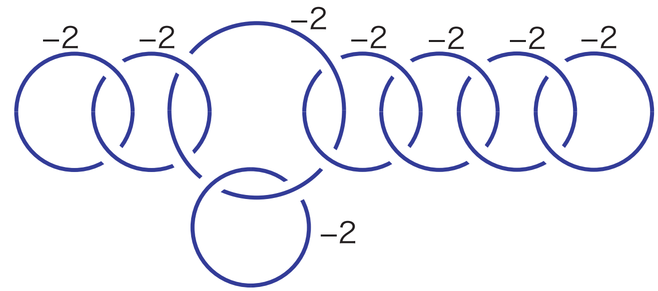


Figure 2-5 : Matrix E_8 and E_8-manifold

$$\partial E_8 = \Sigma(2,3,5) := \{(x,y,z) \in S^5_\epsilon \subset \mathbf{C}^3 \,|x^2 + y^3 + z^5 = 0 \,\}$$ “Poincare sphere”

Exercise. $(S^2 \times S^2)\sharp\overline{\mathbf{C}P^2} \cong \mathbf{C}P^2\sharp\overline{\mathbf{C}P^2}\sharp\overline{\mathbf{C}P^2} \cong (S^2 \tilde{\times} S^2)\sharp\overline{\mathbf{C}P^2}$

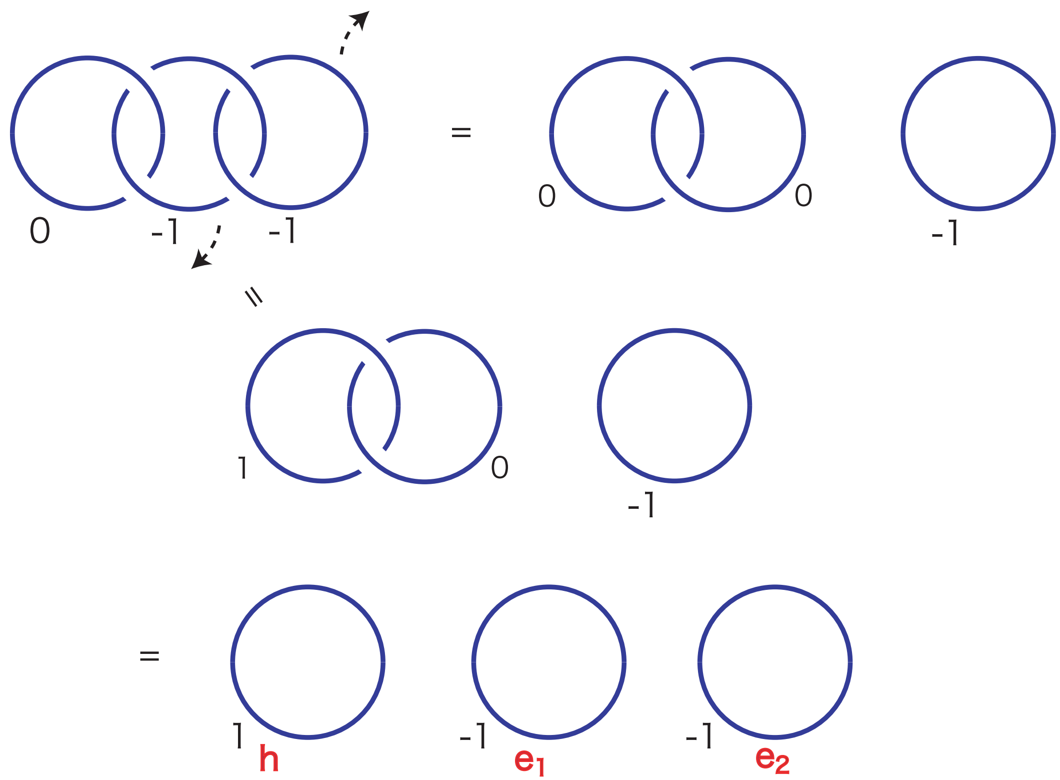


Figure 2-6 :

$$\begin{aligned} H_2(\mathbf{C}P^2\sharp\overline{\mathbf{C}P^2}\sharp\overline{\mathbf{C}P^2};\mathbf{Z}) &= H_2(\mathbf{C}P^2;\mathbf{Z}) \oplus H_2(\overline{\mathbf{C}P^2};\mathbf{Z}) \oplus H_2(\overline{\mathbf{C}P^2};\mathbf{Z}) \\ &=: \mathbf{Z}h \oplus \mathbf{Z}e_1 \oplus \mathbf{Z}e_2. \end{aligned}$$

Exercise. $(S^2 \times S^2)\sharp\overline{\mathbf{C}P^2} \cong \mathbf{C}P^2\sharp\overline{\mathbf{C}P^2}\sharp\overline{\mathbf{C}P^2} \cong (S^2\tilde{\times}S^2)\sharp\overline{\mathbf{C}P^2}$

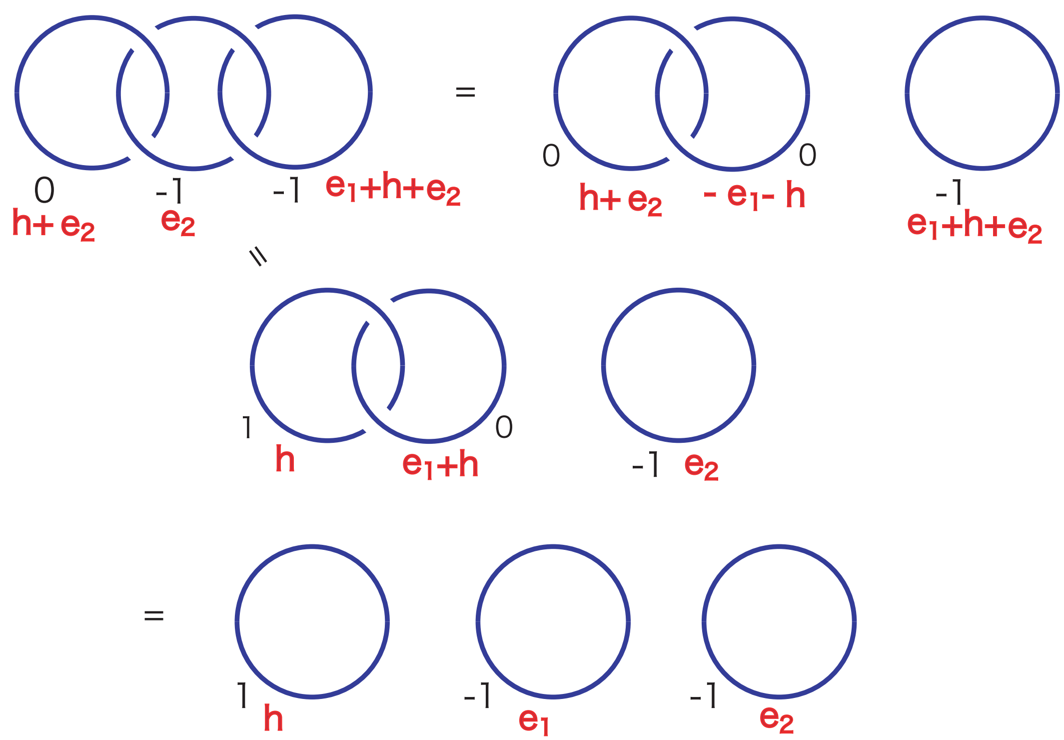


Figure 2-6' :

In $(S^2 \times S^2)\sharp\overline{\mathbf{C}P^2}$, the class $[S^2 \times \{p\}]$ is $h + e_2$, $[\{p\} \times S^2]$ is $-e_1 - h$.
the class $[\mathbf{C}P^1]$ in $\overline{\mathbf{C}P^2}$ is $h + e_1 + e_2$.

§3. 4-dim. with 1-handles

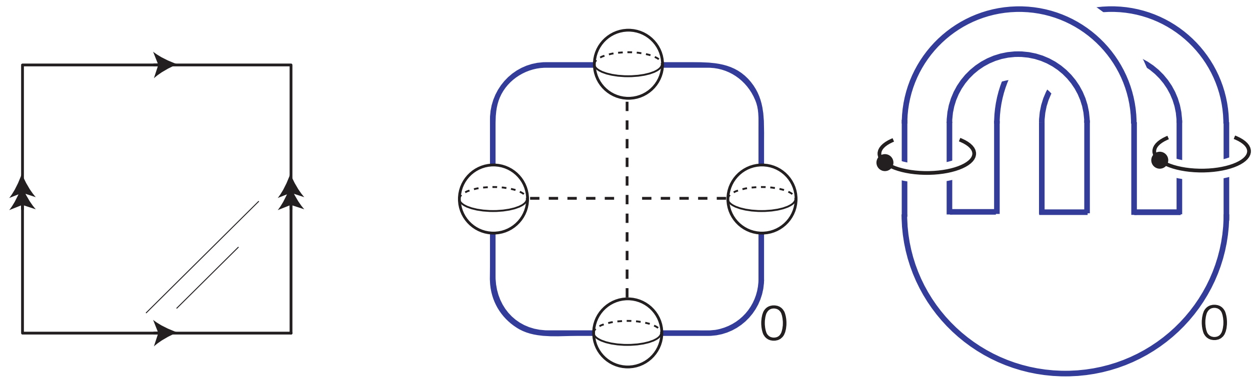


Figure 3-1 : $T^2 \times D^2$

§3. 4-dim. with 1-handles

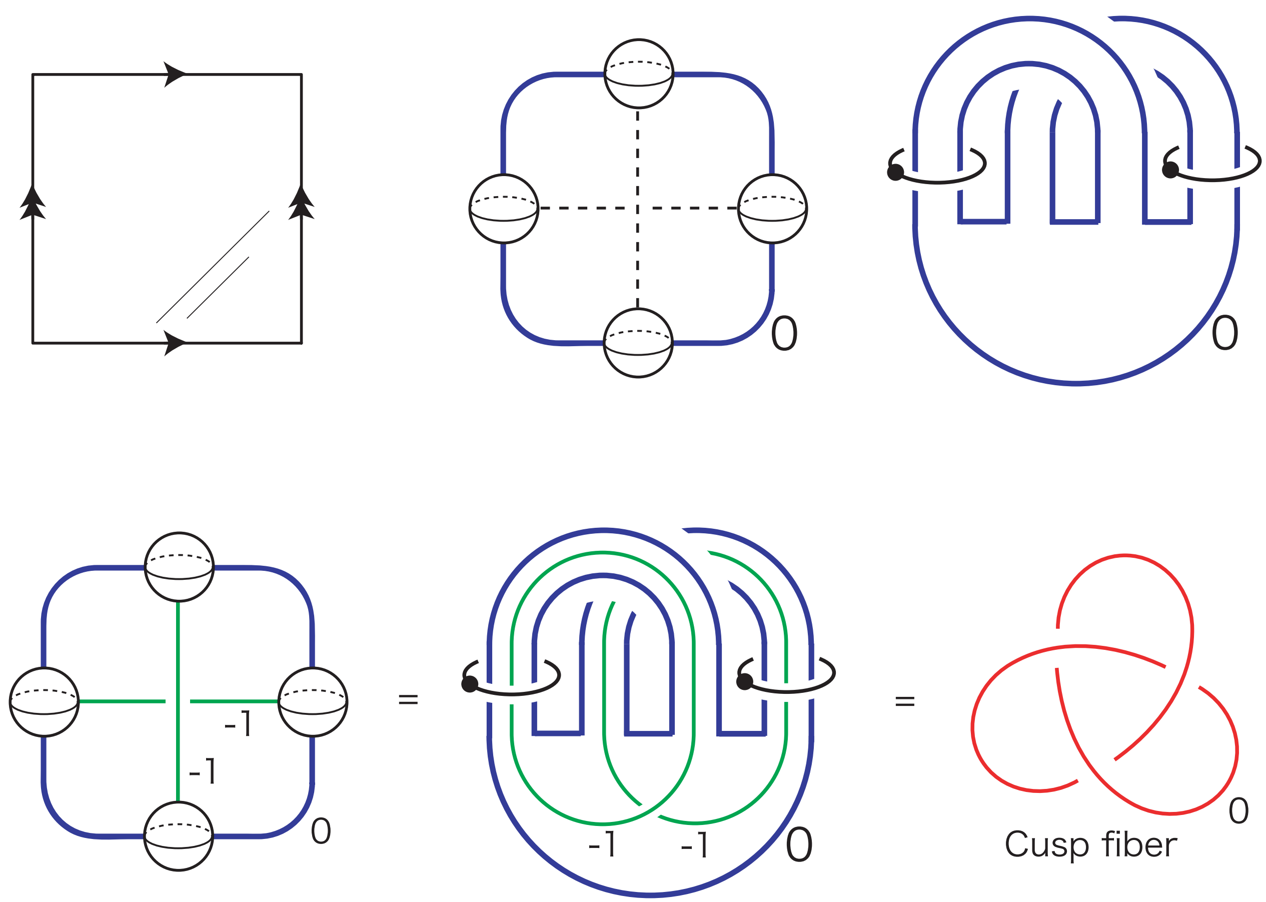
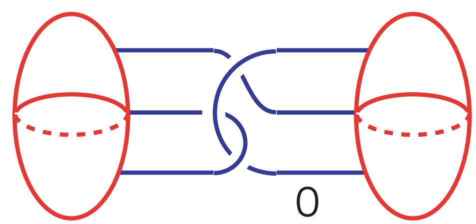
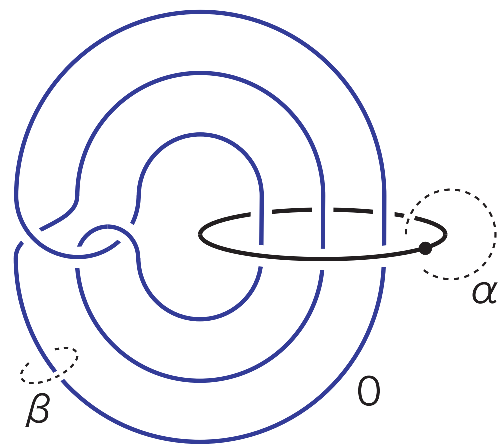


Figure 3-2 : The Cusp fiber

Mazur manifold is contractible, but Not a ball





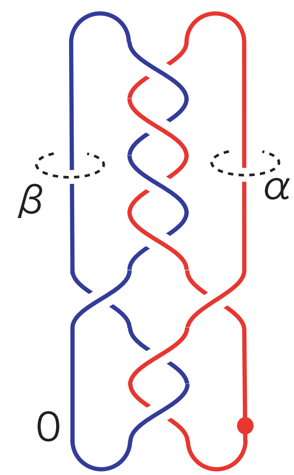


Figure 3-3 : Mazur manifold

Question.

- π_1 of this manifold?, • boundary of it?, ...

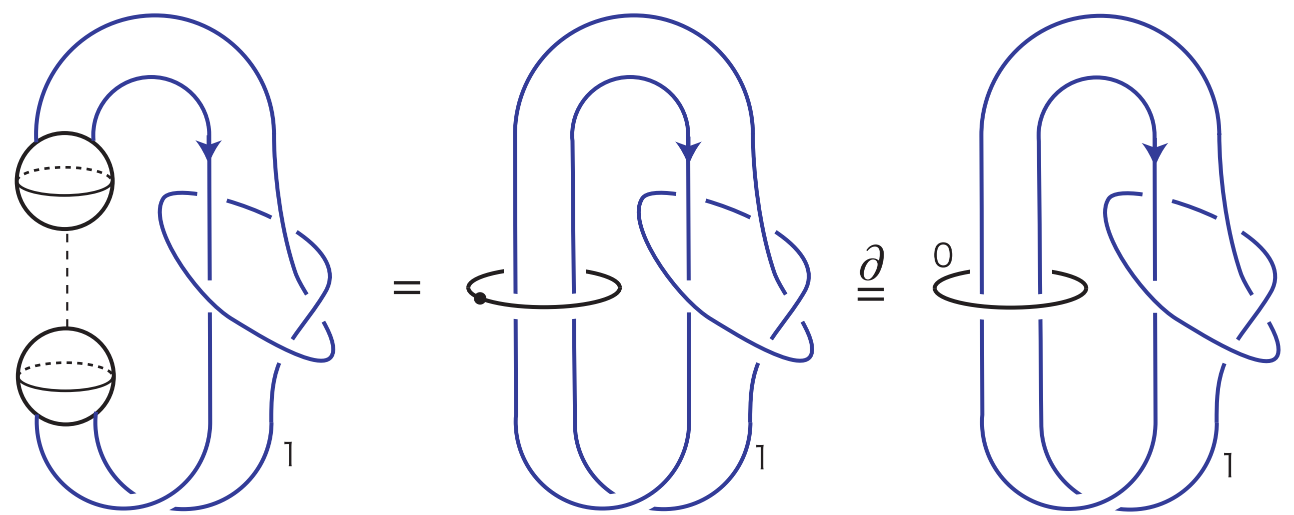


Figure 3-4 : a 4-manifold

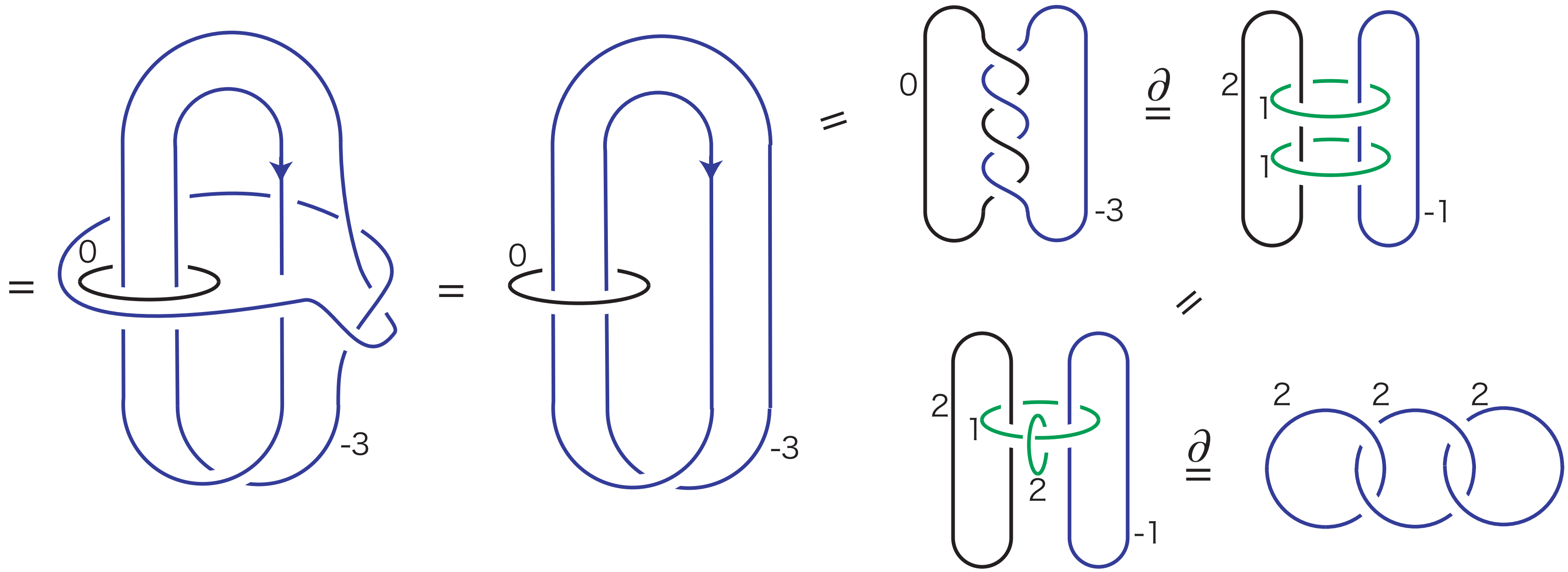
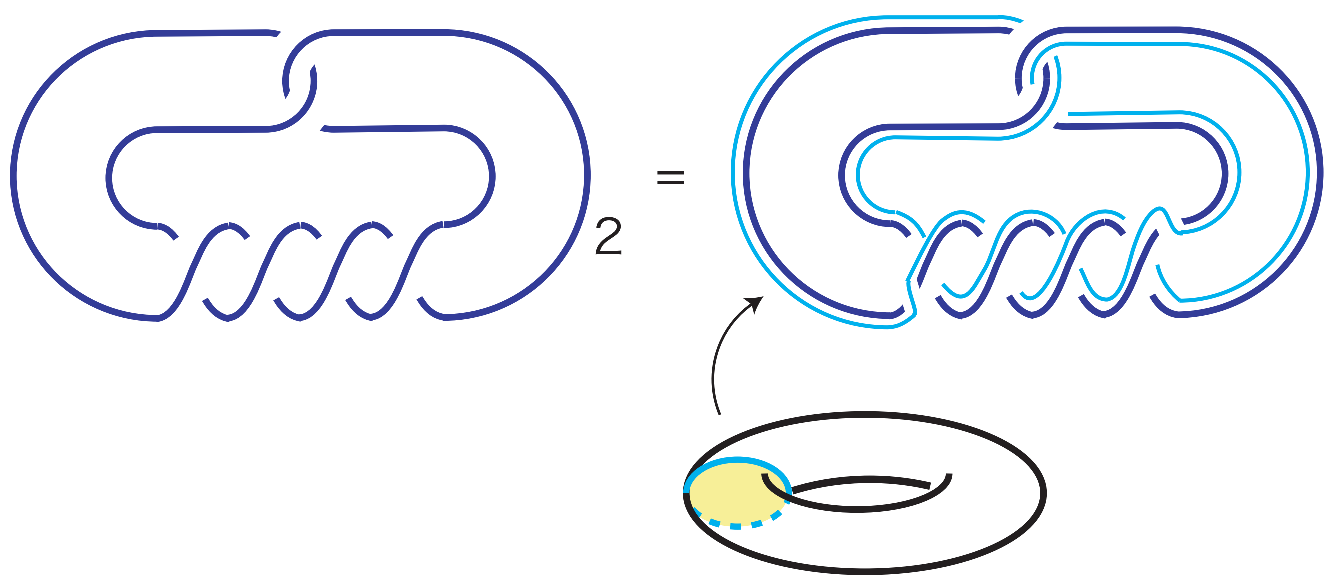


Figure 3-4(2) : the 4-manifold

is a **Q** homology-ball whose $\pi_1 \cong \mathbf{Z}/2\mathbf{Z}$, and boundary is $L(4,1)$.

§4. 3-dim. manifolds, Kirby Calculus

Dehn surgery coefficient = Framing = a parallel curve, or the linking number. *Remove and Reglue* Solid torus along each component such as "the meridian comes to the parallel"



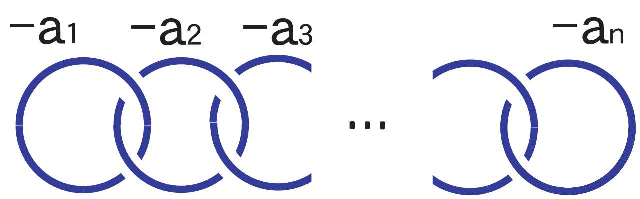
Thm. [Lickorish '62]

Any closed connected oriented 3-manifold is obtained by a framed link L in S^3.

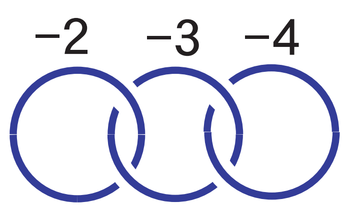
Notation: $M(L)$, or $(K; p)$.

Framed Links for Lens $L(p,q)$

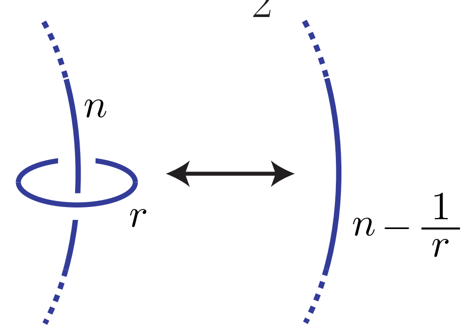
$$\frac{p}{q} = a_1 - \cfrac{1}{a_2 - \cfrac{1}{a_3 - \ddots - \cfrac{1}{a_n}}} \qquad (a_i > 1)$$



$L(18,11)$ $(= L(18,5))$ $\quad \dfrac{18}{11} = 2 - \cfrac{1}{3 - \cfrac{1}{4}}, \quad \dfrac{18}{5} = 4 - \cfrac{1}{3 - \cfrac{1}{2}}$



For $n \in \mathbf{Z}, r \in \mathbf{Q}$



ありがとうございました

最終日まで, じっくり過ごしましょう！